L3.1

Lindblad master equation

$$\frac{d\rho}{dt} = -\frac{i}{\hbar}[H,\rho] + \sum_j \left(L_j \rho L_j^{+} - \frac{1}{2}\{ L_j^{+} L_j , \rho \} \right)$$

spontanous emission, relaxation

$$L = \sqrt{\gamma}\, \sigma_-$$

$$H_0 = \frac{\hbar \omega_0}{2} \sigma_z$$

$$\frac{d}{dt}\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} = -i\omega_0 \begin{pmatrix} 0 & \rho_{01} \\ -\rho_{10} & 0 \end{pmatrix} + \gamma \begin{pmatrix} \rho_{11} & -\frac{1}{2}\rho_{01} \\ -\frac{1}{2}\rho_{10} & -\rho_{11} \end{pmatrix}$$

$$\rho_{00}(t) = \rho_{00}(0) + \rho_{11}(0)\left[1 - e^{-\gamma t}\right]$$

$$\rho_{11}(t) = \rho_{11}(0)\, e^{-\gamma t}$$

$$\rho_{01}(t) = \rho_{01}(0)\, e^{i\omega_0 t}\, e^{-\gamma t/2}$$

$$\rho_{10}(t) = \rho_{10}(0)\, e^{-i\omega_0 t}\, e^{-\gamma t/2}$$

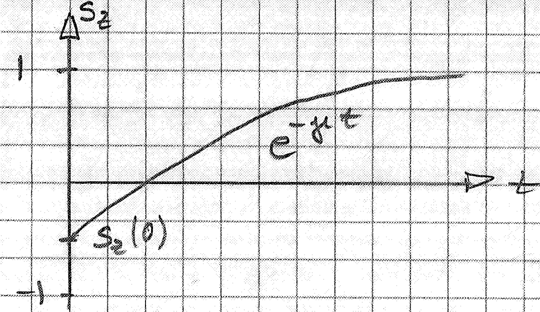
L3.2 $\quad s_z(t) = \langle z \rangle = \mathrm{tr}(\rho\,\sigma_z) = \rho_{00} - \rho_{11}$

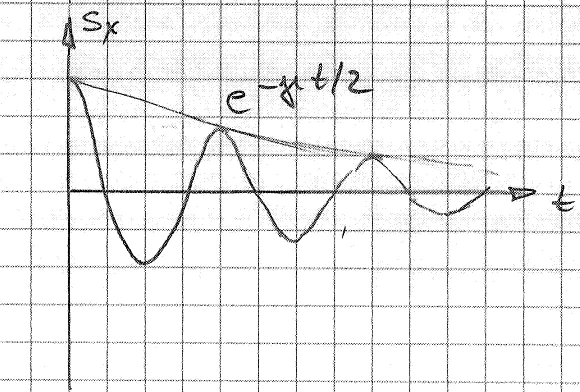
$$s_z(t) = s_z(0)\, e^{-\gamma t} + (1 - e^{-\gamma t})$$

$$s_x(t) = s_x(0) \cos(\omega_0 t)\, e^{-\gamma t/2}$$

$$s_y(t) = s_y(0) \sin(\omega_0 t)\, e^{-\gamma t/2}$$

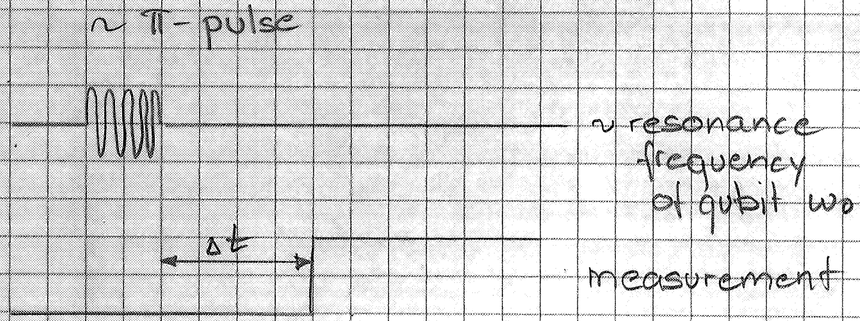
Larmor precession



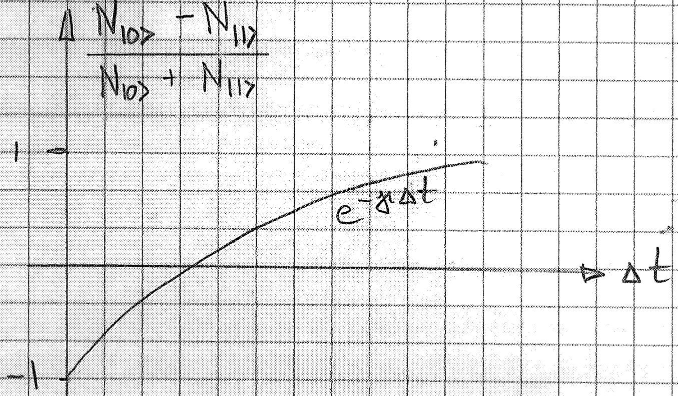


L3.3

How to measure relaxation time



measurement is repeated many times



$N_{|0\rangle}$ number of times measurement resulted $|0\rangle$

$N_{|1\rangle}$ .... $|1\rangle$

Dephasing

$$L_r = \sqrt{\gamma_r}\,\sigma_-$$

$$L_\varphi = \sqrt{\gamma_\varphi}\,\sigma_z$$

$$H_0 = \frac{\hbar\omega_0}{2}\sigma_z$$

$$\frac{d}{dt}\begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix} = i\omega_0 \begin{pmatrix} 0 & \rho_{01} \\ -\rho_{10} & 0 \end{pmatrix} + \gamma_r \begin{pmatrix} \rho_{11} & -\frac{1}{2}\rho_{01} \\ -\frac{1}{2}\rho_{10} & -\rho_{11} \end{pmatrix} + \gamma_\varphi \begin{pmatrix} 0 & -2\rho_{01} \\ -2\rho_{10} & 0 \end{pmatrix}$$

$$\rho_{00}(t) = \rho_{00}(0) + \rho_{11}(0)\left[1 - e^{-t/T_1}\right]$$

$$\rho_{11}(t) = \rho_{11}(0)\, e^{-t/T_1}$$

$$\rho_{01}(t) = \rho_{01}(0)\, e^{i\omega_0 t}\, e^{-t/T_2}$$

$$\rho_{10}(t) = \rho_{10}(0)\, e^{i\omega_0 t}\, e^{-t/T_2}$$

$$\frac{1}{T_1} = \gamma_r \qquad \text{relaxation time}$$

$$\frac{1}{T_2} = \frac{1}{2}\gamma_r + 2\gamma_\varphi \qquad \text{dephasing time}$$

L3.5

$$\frac{1}{T_2} = 2\gamma_\varphi + \frac{1}{2}\frac{1}{T_1}$$

$$T_2 \leq 2T_1$$

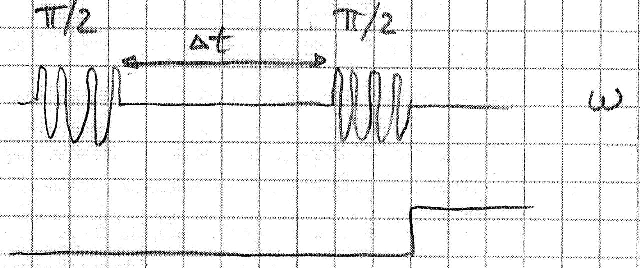
$$\frac{\partial}{\partial t} s_z = -\frac{(s_z - 1)}{T_1}$$

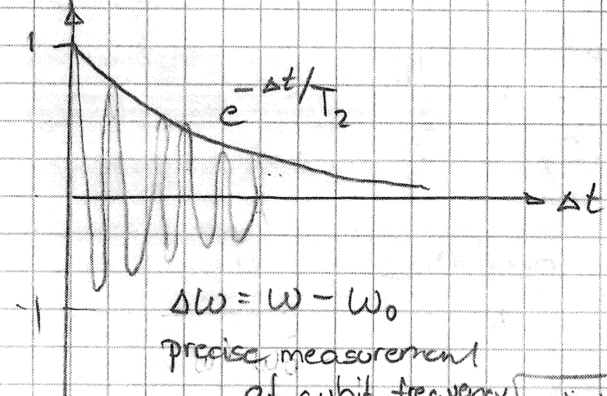
$$\frac{\partial}{\partial t} s_x = -\omega_0 s_y - \frac{s_x}{T_2}$$

$$\frac{\partial}{\partial t} s_y = +\omega_0 s_x - \frac{s_y}{T_2}$$

Bloch equations

Ramsey Fringe





$\Delta\omega = \omega - \omega_0$

precise measurement of qubit frequency

$\boxed{\text{slides}}$

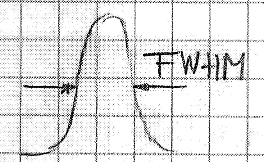
Rabi Oscillations

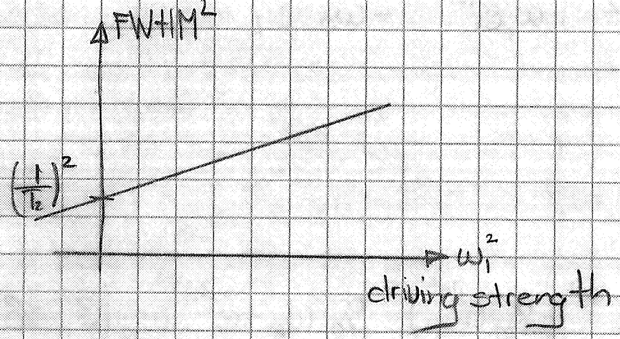
$$H = \frac{\hbar}{2}\begin{pmatrix} \omega_0 & \omega_1 e^{-i\omega t} \\ \omega_1 e^{+i\omega t} & -\omega_0 \end{pmatrix}$$

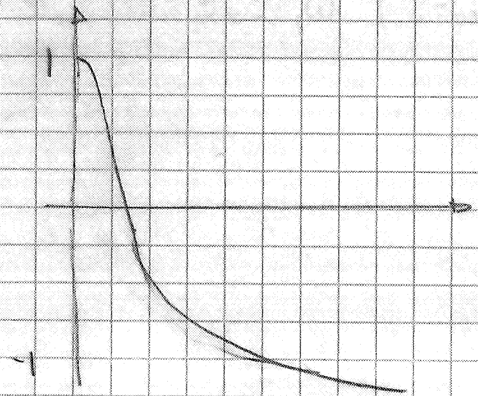
$$[H, \rho] =$$

$$\begin{pmatrix} \omega_1\left(\rho_{10} e^{-i\omega t} - \rho_{01} e^{i\omega t}\right) & \rho_{01}\omega_0 + e^{-i\omega t}\omega_1\left(\rho_{11} - \rho_{00}\right) \\ -\rho_{10}\omega_0 + e^{i\omega t}\omega_1\left(\rho_{00} - \rho_{11}\right) & \omega_1\left(\rho_{01} e^{i\omega t} - \rho_{10} e^{-i\omega t}\right) \end{pmatrix}$$

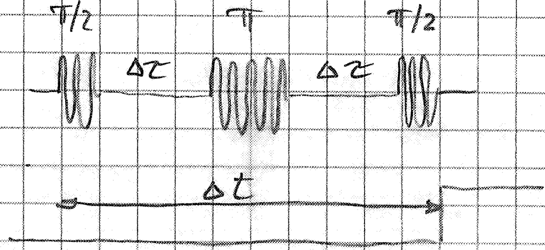
$$L(\omega) = \frac{\omega_1^2}{(\Delta\omega)^2 + \omega_1^2 + \left(\frac{1}{T_2}\right)^2}$$

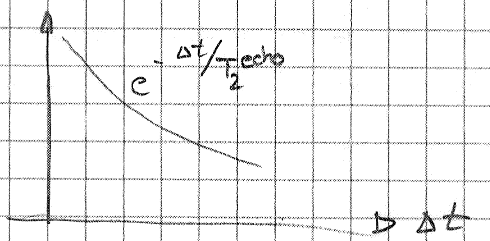






Hahn Echo





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$$\frac{1}{T_2^*} = \frac{1}{T_2} + \frac{1}{T_2^{\text{inhomogenous}}}$$

T_2^{ramsey} , T_2^{echo}